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NORTH CENTRAL FOREST EXPERIMENT STATION, FOREST SERVICE—U.S. DEPARTMENT OF AGRICULTURE

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# A LOCAL NET VOLUME EQUATION FOR IOWA

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## APPLICATION

As a part of the 1974 Forest Survey of Iowa, the Station's Forest Resources Evaluation Research Staff developed a merchantable tree volume equation and tables of coefficients for Iowa. They were developed for both board foot (International ¼-inch rule) and cubic foot volumes, for several species and species groups of growing-stock trees.[1] The equation and the coefficients can be used on a digital computer or modern electronic desk calculator by industrial foresters, consulting foresters, or others in need of quick net-volume-per-tree estimates.

The only data needed to determine tree volumes in Iowa with the equation and coefficients are species, dbh, and site index. However, if site index measurements have not been taken, the average site index for the observations used in constructing tables 1 and 2 can be used.

To use the equation:

1. Select the appropriate table--table 1 for cubic foot volume or table 2 for board foot volume.
2. Obtain the species group number from table 3.
3. Substitute the appropriate coefficients A, B, and C; dbh; and site index from the table into the following equation.

$$\text{Volume} = A \cdot \text{site index} \cdot (1 - e^{-B \cdot \text{dbh}})^{\frac{1}{1-C}}$$

Where e is the well known mathematical constant for the base of the natural or Naperian logarithm = 2.71828. For example the cubic foot volume for a 10.0 inch select white oak on a site with a site index of 55 is:

$$\text{Cubic volume} = 39.90 \cdot 55 (1 - e^{-.012 \cdot 10.0})^{\frac{1}{1-.612}}$$
$$= 7.97$$

## DOCUMENTATION

Data collected on 6,158 trees from 500 plots representing a random sample of forest conditions in the State were used to develop these volume equations. Measurements taken were site index, tree class, dbh, merchantable height, and diameter of the stem outside bark at merchantable height. For sawtimber-size trees (9.0+ inches dbh for softwoods, 11.0+ inches for hardwoods), measurements were taken at both the sawtimber and pulpwood limits of merchantability. We also measured and estimated the volume of cull material (both in cubic feet and board feet) in each tree.

The net volume in these trees was then calculated using equations developed by Robert N. Stone[2] based on S. R. Gevorkiantz's "Composite Volume Tables for the Lake States."[3] Stone's equations compute gross volume in a tree from three measurements--dbh, merchantable height, and diameter of the stem outside bark at merchantable height. The volume is

[1] *All live trees of commercial species except rough and rotten trees.*

[2] *Unpublished data on file at the North Central Forest Experiment Station, St. Paul, Minnesota.*

[3] *S. R. Gevorkiantz and L. R. Olsen. Composite volume tables for timber and their application in the Lake States. Lake States Tech. Bull. 1104, 51 p. 1955.*

Table 1.--*Cubic foot volume regression coefficients for growing stock trees, Iowa, 1974*

$$VOL = A \cdot SI \cdot (1 - e^{-B \cdot dbh})^{\frac{1}{1-C}}$$

| Species group number | Coefficient A | Coefficient B | Coefficient C | Average site index | Observations | Standard error of estimate | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | | | | *No.* | *Cu. Ft.* | |
| 1-3 | 5.18 | 0.039 | 0.685 | 47 | 37 | 3.47 | 0.78 |
| 4 | 39.90 | .012 | .612 | 57 | 1,498 | 7.66 | .87 |
| 5 | 75.96 | .010 | .629 | 60 | 772 | 9.58 | .92 |
| 6 | 5.27 | .032 | .648 | 58 | 12 | 1.48 | .99 |
| 7 | 7.42 | .032 | .679 | 61 | 246 | 8.83 | .91 |
| 8 | 4.83 | .034 | .633 | 58 | 493 | 3.40 | .88 |
| 9 | 94.92 | .005 | .532 | 62 | 217 | 10.05 | .81 |
| 10-11 | 92.79 | .006 | .547 | 81 | 498 | 14.54 | .91 |
| 12 | 4.67 | .045 | .724 | 70 | 206 | 6.84 | .93 |
| 13 | 48.97 | .013 | .664 | 83 | 369 | 23.81 | .90 |
| 14 | 18.48 | .015 | .589 | 69 | 91 | 4.45 | .91 |
| 15 | 121.70 | .007 | .602 | 61 | 335 | 7.07 | .88 |
| 16 | 39.96 | .013 | .624 | 65 | 599 | 8.09 | .92 |
| 17 | 22.32 | .016 | .630 | 63 | 197 | 5.09 | .86 |
| 18 | 125.20 | .011 | .669 | 66 | 64 | 3.97 | .86 |
| 19 | 3.18 | .051 | .716 | 66 | 101 | 10.00 | .88 |
| 20 | 2.28 | .058 | .714 | 71 | 175 | 4.32 | .91 |
| 21 | 23.63 | .013 | .602 | 73 | 76 | 8.59 | .79 |
| 22 | 39.67 | .010 | .604 | 67 | 172 | 11.19 | .86 |

Table 2.--*Board foot volume regression coefficients for growing stock trees, Iowa, 1974*

| Species group number | Coefficient A | Coefficient B | Coefficient C | Average site index | Observations | Standard error of estimate | $R^2$ |
|---|---|---|---|---|---|---|---|
| | | | | | *No.* | *Bd. Ft.* | |
| 1-3 | 13.57 | 0.050 | 0.671 | 46 | 14 | 26.29 | 0.55 |
| 4,6 | 100.10 | .014 | .583 | 57 | 980 | 61.16 | .69 |
| 5 | 44.55 | .026 | .632 | 61 | 586 | 66.68 | .81 |
| 7 | 23.32 | .050 | .734 | 59 | 137 | 99.70 | .73 |
| 8 | 20.51 | .035 | .604 | 58 | 184 | 34.35 | .62 |
| 9 | 8.42 | .071 | .690 | 63 | 139 | 78.31 | .47 |
| 10-11 | 12.95 | .059 | .744 | 81 | 352 | 96.04 | .77 |
| 12 | 10.12 | .096 | .846 | 72 | 108 | 69.25 | .75 |
| 13 | 106.52 | .020 | .664 | 83 | 329 | 149.61 | .84 |
| 14 | 79.77 | .016 | .590 | 68 | 41 | 39.68 | .76 |
| 15 | 19.80 | .049 | .709 | 61 | 209 | 60.36 | .65 |
| 16 | 21.23 | .045 | .707 | 63 | 319 | 72.84 | .76 |
| 17 | 46.94 | .024 | .642 | 64 | 107 | 40.72 | .68 |
| 18 | 7.41 | .086 | .777 | 66 | 26 | 43.23 | .28 |
| 19 | 15.73 | .038 | .617 | 68 | 56 | 86.80 | .61 |
| 20 | 9.46 | .061 | .713 | 73 | 93 | 43.56 | .66 |
| 21 | 334.23 | .006 | .553 | 74 | 47 | 43.03 | .76 |
| 22 | 40.05 | .021 | .599 | 68 | 90 | 63.96 | .82 |

then corrected for differences in bark thickness between species and the field estimate of cull volume is subtracted to arrive at net volume. If tree heights are tallied while making a local cruise, measure bark thickness at the same time and use either the tables from Gevorkiantz and Olsen (1955) or Stone's equation, instead of the equations presented here, to compute volume.

Experience in working with volume-dbh relations suggested that the best mathmatical model for this type of data was the Gomperz function: $Y = S \cdot B^{C^{D \cdot X}}$, $S = A \cdot SI$, and A, B, C and D are parameters. This is a special form of the more general Chapman-Richards function: $Y = S(1-e^{-B \cdot X})^{\frac{1}{1-C}}$, $S = A \cdot SI$, and A, B and C are parameters. In the past the Chapman-Richards function was difficult to fit for volume-dbh data because the parameter C tended to approach the value 1. However, a new fitting procedure enabled us to use the Chapman-Richards function and solve for 3 instead of the 4 parameters of the Gomperz function. In the function, the dependent variable Y is net tree volume, the independent variable X is dbh, and the asymptote S is a function of site index ($S = A \cdot SI$).

The species groups used in the tables were primarily determined by the number of observations available for a group (a minimum number of observations are necessary to form an equation using least-squares estimates), and similarity of bole form of the species within a group. The number of observations, the standard error of estimate, and the $R^2$ for each regression estimate are shown in tables 1 and 2.

Table 3.--*Species and species group numbers for Iowa trees*

| Species or species group | Scientific name | Number |
|---|---|---|
| Softwoods | | |
| Red and white pine | *Pinus resinosa*<br>*Pinus strobus* | 1 |
| Eastern redcedar | *Juniperus virginiana* | 2 |
| Other softwoods | | 3 |
| Hardwoods | | |
| Select white oaks | *Quercus alba*<br>*Quercus bicolor*<br>*Quercus macrocarpa*<br>*Quercus michauxii*<br>*Quercus muehlenbergii* | 4 |
| Select red oaks | *Quercus falcata* var. *pagodaefolia*<br>*Quercus rubra* | 5 |
| Other white oaks | *Quercus lyrata*<br>*Quercus prinus*<br>*Quercus stellata* var. *stellata* | 6 |
| Other red oaks | *Quercus coccinea*<br>*Quercus falcata*<br>*Quercus imbricaria*<br>*Quercus marilandica*<br>*Quercus nigra*<br>*Quercus palustris*<br>*Quercus phellos*<br>*Quercus velutina* | 7 |

(Table 3 cont. on next page)

(Table 3 cont.)

| Species or species group | Scientific name | Number |
|---|---|---|
| Hickory | *Carya cordiformis*<br>*Carya glabra*<br>*Carya illinoensis*<br>*Carya laciniosa*<br>*Carya ovata*<br>*Carya tomentosa* | 8 |
| Hard maple | *Acer nigrum*<br>*Acer saccharum* | 9 |
| Soft maple | *Acer rubrum* var. *rubrum* | 10 |
| Tupelo and blackgum | *Nyssa aquatica*<br>*Nyssa sylvatica* var. *sylvatica*<br>*Nyssa sylvatica* var. *biflora* | 11 |
| Ash | *Fraxinus americana*<br>*Fraxinus nigra*<br>*Fraxinus pennsylvanica*<br>*Fraxinus quadrangulata* | 12 |
| Cottonwood | *Populus deltoides*<br>*Populus heterophylla* | 13 |
| Aspen | *Populus grandidentata*<br>*Populus tremuloides* | 14 |
| Basswood | *Tilia americana* | 15 |
| Elm | *Ulmus alata*<br>*Ulmus americana*<br>*Ulmus rubra*<br>*Ulmus thomasii* | 16 |
| Black walnut | *Juglans nigra* | 17 |
| Black cherry | *Prunus serotina* | 18 |
| Hackberry | *Celtis occidentalis* | 19 |
| Willow | *Salix nigra* | 20 |
| Birch | *Betula alleghaniensis*<br>*Betula nigra*<br>*Betula papyrifera* var. *papyrifera* | 21 |
| Other hardwoods | *Acer negundo*<br>*Aesculus glabra*<br>*Diospyros virginiana*<br>*Gleditsia triacanthos*<br>*Gymnocladus dioicus*<br>*Juglans cinerea*<br>*Maclura pomifera*<br>*Morus rubra*<br>*Platanus occidentalis*<br>*Robinia pseudoacacia*<br>*Sassafras albidum* | 22 |
| Noncommercial species | | 23 |



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